# Mesonic Spectrum from a Dynamical Gravity/Gauge model

W. de Paula<sup>a\*</sup> and T. Frederico<sup>a</sup>.

<sup>a</sup>Dep. de Física, Instituto Tecnológico de Aeronáutica, CTA, 12228-900. São José dos Campos, Brazil

Within a formulation of a Dynamical AdS/QCD model we calculate the spectrum of light flavored mesons. The background fields of the model correspond to an IR deformed Anti de Sitter metric coupled to a dilaton field. Confinement comes as a consequence of the dilaton dynamics coupled to gravity. Additionally to the Regge-like spectrum of light- scalar, vector and higher spin mesons, we obtain the decay width of scalar mesons into two pions.

### 1. Introduction

The non perturbative aspects of strong interaction are extremely difficult to treat analytically. Most of our knowledge about the low energy limit of the strong force comes from Lattice calculations. In this context, the AdS/CFT correspondence [1] represents an attractive alternative to investigate non perturbative aspects of gauge theories. This duality connects the string theory amplitudes on asymptotically AdS  $\times X$  spacetime into gauge invariant, local operators of a conformal field theory (CFT). As a consequence, the notoriously complex strong-coupling regime of large- $N_c$  gauge theories can be approximated (in low-curvature regions) by weakly coupled and hence analytically treatable classical gravities.

Extensions of this idea to Quantum Chromodynamics (QCD) either start from specific D-brane setups in ten- (or five-) dimensional supergravity [2,3,4,5,6] and derive the corresponding gauge theory properties, or try to guess a suitable background and to improve it in bottom-up fashion by comparing the predictions to QCD data. The first bottom-up model (Hard Wall) developed by Polchinsky and Strassler [7] shows that the conformal invariance of  $AdS_5$  in the UV limit implements the counting rules which govern the scaling behavior of hard QCD scattering amplitudes. An infrared cutoff on the fifth dimension at the QCD scale  $\Lambda_{QCD}$  gives the mass gap and a discrete hadron spectrum. This model reproduces

a huge amount of hadron phenomenology[8]. On the other hand it does not reproduce Regge trajectories on the mass spectrum  $(M^2 \times n)$ . To correct this shortcoming Karch, Katz, Son and Stephanov developed the Soft-Wall model[9]. In this approach the AdS<sub>5</sub> geometry is kept intact while an additional dilaton background field is responsible for the conformal symmetry breaking. This dilaton soft-wall model indeed generates linear Regge trajectories  $m_{n,S}^2 \sim n + S$  for light-flavor mesons of spin S and radial excitation n. (Regge behavior can alternatively be encoded via IR deformations of the AdS<sub>5</sub> metric [10,11].)

However, the resulting vacuum expectation value (vev) of the Wilson loop in the dilaton soft wall model does not exhibit the area-law behavior which a linearly confining static quark-antiquark potential would generate. It happens because the model uses an AdS metric which is not of a confining type by the Wilson Loop analysis [12,13]. In addition the soft wall model background is not a solution of a dual classical gravity theory. Therefore, one has to impose all gauge theory vacuum properties (confinement, chiral symmetry breaking and condensates) in an ad-hoc manner, and the desired connection to the dynamics of a QCD dual remains untouched [14].

Csaki and Reece [15] analyzed the solutions of a 5d dilaton-gravity Einstein equations (see also[18]) using the formalism of superpotential. Their conclusion is that it would not be possible to solve those equations and obtain a linear confining background. They also suggest that

<sup>\*</sup>Supported by CAPES

it would be possible to get a solution by analyzing a tachyon-dilaton-graviton model. This idea was successfully implemented by Batell and Gherghetta [16].

We took an alternative route and we showed [17] that a linear confining background is possible as a solution of the dilaton-gravity coupled equations in a deformed AdS model. It means that our solution allows to obtain a spectrum of high-spin mesons very close to Regge trajectories for the lower excited states (where we have experimental data) and an exact linear Regge trajectory for very high excitations. We solve self-consistently the dilaton-gravity model, i.e., we adopt an active dilaton in contrast to the Soft-Wall model where a passive dilation was considered. Our model belongs to the general class of "Improved AdS/QCD theories" proposed recently by Gürsoy, Kiritsis and Nitti [18].

# 2. Dynamical AdS/QCD model

In this section we will make a review of the Dynamical AdS/QCD model as proposed by de Paula, Frederico, Forkel and Beyer [17]. Let's take the action for a five-dimensional gravity coupled to a dilaton field:

$$S = \int \frac{d^5x}{2\kappa^2} \sqrt{g} \left( -R - V(\Phi) + \frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi \right),$$

where  $\kappa$  is the Newton constant in 5 dimensions and  $V(\Phi)$  is the scalar-field potential . We will be restricted to a metric family given by:

$$g_{MN} = e^{-2A(z)}\eta_{MN},$$

where  $\eta_{MN}$  is the Minkowski metric.

Minimizing the action we obtain the coupled Einstein equations

$$6A^{\prime 2} - \frac{1}{2}\Phi^{\prime 2} + e^{-2A(z)}V(\Phi) = 0, \qquad (1)$$

$$-3A'^{2} + 3A'' - \frac{1}{2}\Phi'^{2} - e^{-2A(z)}V(\Phi) = 0, \qquad (2)$$

$$\Phi'' - 3A'\Phi' - e^{-2A(z)}\frac{dV}{d\Phi} = 0.$$
 (3)

See that we can determine the dilaton field directly from the metric model as:

$$\Phi' = \sqrt{3A'^2 + 3A''} \ ,$$

where we choose the positive sign for the root without loosing generality. Substituting the dilaton field in equation (1) we obtain the dilaton potential as

$$V(\Phi) = \frac{3e^{2A}}{2} \left( A'' - 3A'^2 \right),\,$$

by solving the dilaton-gravity 5d Einstein equations.

#### 2.1. Hadronic Resonances

As we have defined the dilaton-metric background of the model, we are now able to calculate the meson mass spectrum in the spirit of AdS/QCD duality. To do so, we utilize the AdS/CFT dictionary in the sense that for each operator in the 4d gauge theory there is a field propagating in the bulk. For definiteness we follow the notation of ref. [9]. The 5d action for a gauge field  $\phi_{M_1...M_S}$  of spin S in the background is given by

$$I = \frac{1}{2} \int d^5 x \sqrt{g} e^{-\Phi} \left( \nabla_N \phi_{M_1 \dots M_S} \nabla^N \phi^{M_1 \dots M_S} \right).$$

As in [9] and [19], we utilize the axial gauge. To this end, we introduce new spin fields  $\widetilde{\phi}_{...} = e^{2(S-1)A}\phi_{...}$ . In terms of this new field, the action is then given by

$$I = \frac{1}{2} \int d^5x e^{-5A} e^{-\Phi} e^{-4(S-1)A} e^{2A(S+1)} \times \partial_N \widetilde{\phi}_{M_1...M_S} \partial_N \widetilde{\phi}_{M_1...M_S} . \tag{4}$$

Using (4) the equation for the modes  $\widetilde{\phi}_n$  of the higher spin field  $\widetilde{\phi}$  is derived, viz.

$$\partial_z \left( e^{-B} \partial_z \tilde{\phi_n} \right) + m_n^2 e^{-B} \tilde{\phi_n} = 0,$$

where  $B = A(2S - 1) + \Phi$ . Via the substitution  $\tilde{\phi}_n = e^{B/2}\psi_n$ , one obtains a Sturm-Liouville equation

$$\left(-\partial_z^2 + \mathcal{V}_{eff}(z)\right)\psi_n = m_n^2 \psi_n,$$

where the B dependent term in this equation may be interpreted as an effective potential for the string mode, written as

$$\mathcal{V}_{eff}(z) = \frac{B'^2(z)}{4} - \frac{B''(z)}{2}.$$

Hence, for each metric A and dilaton field  $\Phi$  we get

$$\mathcal{V}_{eff}(z) = A^{\prime 2} + \frac{5}{4}A^{\prime \prime} - \sqrt{3} \, \frac{A^{\prime \prime \prime} + 4A^{\prime}A^{\prime \prime} + 2A^{\prime 3}}{4\sqrt{A^{\prime 2} + A^{\prime \prime}}}$$

$$+S^2A'^2 + S\left(A'\sqrt{3A'^2 + 3A''} - A'^2 - A''\right)$$
,

consistent with the solutions of the Einstein equations. By solving the eigenvalue mode equation, we obtain a mass spectrum  $m_n^2$  starting from the effective potential. Due to the gauge/gravity duality this mass spectrum corresponds to the mesonic resonances in the 4d space-time.

#### 2.2. Scalar Mesons

For scalars we can repeat the same derivation as we did in the last section except for the fact that for these mesons obviously we do not have to make any gauge choice. In particular scalar mesons were analyzed in [20,21]. However, both works do not include the sigma.

We start from the action [22]

$$I = \int \frac{d^4x}{2} dz \sqrt{|g|} \left( g^{\mu\nu} \partial_{\mu} \varphi(x,z) \partial_{\nu} \varphi(x,z) - m_5^2 \varphi^2 \right),$$

where  $m_5^2 = M_5^2/\Lambda_{QCD}^2$ , that describes a scalar mode propagating in the dilaton-gravity background. We factorize in terms of the holographic coordinate  $\varphi(x,z) = e^{iP_\mu x^\mu} \varphi(z)$ ,  $P_\mu P^\mu = m^2$ .

The string modes of the massive scalar field  $\varphi$  can be rewritten in terms of the reduced amplitudes  $\psi_n(z) = \varphi_n(z) \times e^{-(3A+\Phi)/2}$  which satisfy the Sturm-Liouville equation, as we have written before for the spin states. The scalar string-mode potential given by

$$\mathcal{V}(z) = \frac{B^{2}(z)}{4} - \frac{B^{2}(z)}{2} + \frac{M_{5}^{2}}{\Lambda_{QCD}^{2}} e^{-2A(z)}, \quad (5)$$

with  $B = 3A + \Phi$ . The gauge/gravity dictionary identifies the eigenvalues  $m_{n,S}^2$  with the squared meson mass spectrum of the boundary gauge theory.

The AdS/CFT correspondence states that the wave function should behave as  $z^{\tau}$ , where  $\tau = \Delta - \sigma$  (conformal dimension minus spin) is the twist dimension for the corresponding interpolating operator that creates the given state configuration [7]. The five-dimensional mass chosen as [23]  $M_5^2 = \tau(\tau - 4)$ , fixes the UV limit of the dual string amplitude with the twist dimension.

#### 3. Analytical Analysis

In order to present an analytical view of confinement from the gravity model, as have been done in refs. [18,17], we will focus on a very simple polynomial metric, representing an IR deformation of AdS metric:

$$A(z) = \log(z) + z^{\lambda} + \dots, \tag{6}$$

where  $\lambda$  is a real parameter. Our units are such that the AdS<sub>5</sub> radius is unity. The first term reflects the AdS metric that dominates the UV limit. The second term is the leading one in the IR region and any subleading term is irrelevant for the present discussion of the Regge trajectory for the high excited string states dual to mesons. The dilaton field is obtained by integrating Eq. (2) with the boundary condition  $\Phi(0) = 0$ . In particular one gets, for  $z \to 0$  and  $z \to \infty$ , respectively

$$\Phi(z) \sim c_0 z^{\frac{\lambda}{2}}$$
 and  $\Phi(z) \sim c_\infty z^{\lambda}$ ,

where  $c_0 = 2\sqrt{3(1+1/\lambda)}$  and  $c_\infty = \sqrt{3}$ . The dilaton potential for  $z \to 0$  is

$$V(\Phi) \sim -6 + \frac{3}{2c_0^2}(\lambda + 1)(\lambda - 8)\Phi^2$$
,

and for  $z \to \infty$  we obtain:

$$V(\Phi) \sim -\frac{9}{2c_{\infty}^2} \lambda^2 \Phi^2 e^{2\Phi/c_{\infty}}$$
,

which diverges exponentially reflecting the exponential form of the metric model. As an example, let us discuss the UV and IR properties of the effective potential for nonzero spin mesons. For small values of z it can be easily expanded giving:

$$\mathcal{V}_{eff}(z) = \frac{S^2 - \frac{1}{4}}{z^2} + \sqrt{3(\lambda^2 + \lambda)} \left( S - \frac{\lambda}{4} \right) z^{\frac{\lambda}{2} - 2} + \frac{\lambda}{4} \left( 8S^2 - S(4\lambda + 4) + 5\lambda + 3 \right) z^{\lambda - 2} + \dots, \quad (7)$$

which shows a spin dependence in all lower order terms. In the IR limit the metric (6) leads to the following effective potential

$$\mathcal{V}_{eff}(z) \to \frac{\lambda^2}{4} (2S - 1 + \sqrt{3})^2 z^{2\lambda - 2},$$
 (8)

which presents a discrete spectrum for the normalizable string modes if  $\lambda > 1$ . It is worthwhile to point out that the analysis of the effective potential gives a constraint for a confining metric consistent with the one found in the analysis of the Wilson loop (see [18]).

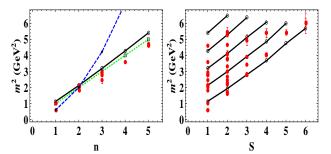


Figure 1. Radial excitations of the rho meson in the hard-wall (dashed line), soft-wall [9] (dotted line) and our dynamical soft-wall (solid line, for  $\Lambda_{QCD}=0.3$  GeV) backgrounds(left panel). Dynamical AdS/QCD squared mass predictions of spin excitations (right panel). Experimental data from [24].

# 4. Phenomenological Results

In our derivations we reduced the problem of modelling hadronic resonances by solving a Sturm-Liouville equation with a given potential (5 or 8). The amazing point is that the effective potential depends only on the metric, which automatically constructs a self consistent dilaton-gravity background. Therefore our modelling is at the level of proposing a metric ansatz that generates the experimental data available (mass, decay constants, form factors,...). In addition this metric has to satisfy the following conditions: i) in the UV it has to become AdS, because QCCD is conformal in this limit and we have to recover the Maldacena duality; ii) in the IR the warp factor has to have a polynomial power of  $\lambda = 2$ , in order to have confinement by the Wilson Loop criteria and linear Regge trajectories. Our ansatz is:

$$A(z) = Log(\xi z \Lambda_{QCD}) + \frac{(\xi z \Lambda_{QCD})^2}{1 + e^{(1 - \xi z \Lambda_{QCD})}} \ ,$$

where  $\xi$  is a scale transformation that connects the gravity background in which different string modes dual propagate in the holographic coordinate. For scalars  $\xi=0.58$ . To distinguish the pion states in our model, the fifth dimensional mass was rescaled according to  $M_5^2 \to M_5^2 + \lambda z^2$  (see [11]). The constraints are the pion mass, the slope of the Regge trajectory and the twist 2 from the operator  $\bar{q}\gamma^5 q$ . The

Figure 2. Regge trajectory for  $f_0$  (left panel) and pion (right panel) from the Dynamical AdS/QCD model with  $\Lambda_{QCD}=0.3$  GeV. Experimental data from [24].

results for the pion Regge trajectory are shown in figure 2 for  $\xi=0.88$  and  $\lambda=-2.19 {\rm GeV}^2$ . For high-spin mesons we have an equation to obtain the scale factor  $\xi=S^{-0.3329}$  (in particular, see that in our previous work [17] we also have the Regge trajectories with a different metric ansatz). With the present metric ansatz we obtain the Regge trajectories in agreement to experimental data for vector, high-spin, scalar and pseudoscalar mesons.

## 5. Decay Amplitudes

The  $f_0$ 's partial decay width into  $\pi\pi$  are calculated from the overlap integral of the normalized string amplitudes (Sturm-Liouville form) in the holographic coordinate dual to the scalars  $(\psi_n)$  and pion  $(\psi_\pi)$  states,

$$h_n = k \int_0^\infty dz \ \psi_\pi^2(z) \psi_n(z) \ , \tag{9}$$

where k is a constant with dimension  $\sqrt{mass}$  fitted to the experimental value of the  $f_0(1500) \to \pi\pi$  partial decay width. The overlap integral is our guess for the dual representation of the transition amplitude  $S \to PP$  and therefore the decay width is given by  $\Gamma^n_{\pi\pi} = \frac{1}{8\pi}|h_n|^2 \frac{p_\pi}{m_n^2}$ , where  $p_\pi$  is the pion momentum in the meson rest frame. The Sturm-Liouville amplitudes of the scalar (pseudoscalar) modes are normalized just as a bound state wave function in quantum mechanics [25,26], which also corresponds to a normalization of the string amplitude

$$\int_0^\infty dz \psi_m(z) \psi_n(z) = \delta_{mn} \ .$$

Table 1 Two-pion decay width and masses for the  $f_0$  family. Experimental values from Particle Data Group [24]. <sup>†</sup>Mixing angle of  $20^o$ . \*Fitted value.

Meson	$M_{exp}(\mathrm{GeV})$	$M_{th}({ m GeV})$	$\Gamma_{\pi\pi}^{exp}({ m MeV})$	$\Gamma^{th}_{\pi\pi}({ m MeV})$
$f_0(600)$	0.4 - 1.2	0.86	600 - 1000	535
$f_0(980)$	$0.98 \pm 0.01$	1.10	$\sim \! 15  80$	$42^\dagger$
$f_0(1370)$	1.2 - 1.5	1.32	$\sim$ 41-141	141
$f_0(1500)$	$1.505 \pm 0.006$	1.52	$38 \pm 3$	38*
$f_0(1710)$	$1.720 {\pm} 0.006$	1.70	$\sim 0$ -6	5
$f_0(2020)$	$1.992 {\pm} 0.016$	1.88	_	0.0
$f_0(2100)$	$2.103 \pm 0.008$	2.04	_	1.2
$f_0(2200)$	$2.189 {\pm} 0.013$	2.19	_	2.5
$f_0(2330)$	2.29 - 2.35	2.33	_	2.8

The two-pion partial decay width for the  $f_0$ 's present in the particle listing of PDG, are calculated with Eq. (9) and shown in Table I. The width of  $f_0(1500)$  is used as normalization for the parameter . In particular for  $f_0(600)$  the model gives a width of about 500 MeV, while its mass is 860 MeV. A large range of experimental values is quoted in PDG for the sigma mass and width (see Table I).

The E791 experiment quotes  $m_{\sigma} = 478^{+24}_{-23} \pm 17$  MeV and  $\Gamma_{\sigma} = 324^{+42}_{-40} \pm 21$  MeV [27], which in has a width consistent with our model while the mass appears somewhat larger. The CLEO collaboration [28] quotes  $m_{\sigma} = 513\pm32$  MeV and  $\Gamma_{\sigma} = 335\pm67$  MeV, and a recent analysis of the sigma pole in the  $\pi\pi$  scattering amplitude from ref.[29] gives  $m_{\sigma} = 441^{+18}_{-10}$  MeV and  $\Gamma_{\sigma} = 544^{+18}_{-25}$  MeV. Other analysis of the  $\sigma$ -pole in the  $\pi\pi \to \pi\pi$  scattering amplitude present in the decay of heavy mesons indicates a mass around 500 MeV [30].

# 6. Conclusions

In this paper we presented a Dynamical AdS/QCD model applied to light meson spectroscopy. We solved the dilaton-gravity coupled equations within a metric model and obtained a linear confining background for the string modes dual to mesons. We obtained a spectrum of the light-favored high-spin, scalar and pseudoscalar mesons in agreement with the experimental data. In addition we calculated the decay amplitude for the  $f_0$ 's into two pions to further check the consistency of the physical scales of the model, as this quantity is strongly sensitive to the relative size of the different mesons.

In particular, the  $f_0(980)$  is identified with the first excitation of the string model dual to  $q\bar{q}$  state (see Table I). We interpret this shift to a value above the experimental value, i.e., 1.1 GeV compared to 0.98 GeV as due to a rescaling of the string mass as in the pion case, that also should be the case for the sigma. By increasing the excitation of the scalar meson this shift tends to decrease (see  $f_0(1500)$  in Table I). We observed that the experimental value of  $\Gamma_{\pi\pi}$  for  $f_0(980)$  is too small compared to our result. This indicated that a strong mixing, e.g., of  $s\bar{s}$  with light non-strange quarks[31], should be present in the model. To account for that a mixing angle for  $f_0(980)$  of  $\pm 20^\circ$  was obtained from the partial width, and values between  $\sim 12^\circ$  to  $28^\circ$  fits  $\Gamma^{\pi\pi}$  within the experimental range.

Let us remind an interesting observation, that the string mode amplitude could be identified with the valence light-front wave function as pointed out by Brodsky and de Téramond [32]. The wave equation in the Sturm-Liouville form is identified with the squared mass operator eigenvalue equation for the valence component of the meson light-front wave function (see also [33]). An alternative way to reproduce Regge trajectories using the anomalous dimension of the operators are given by Vega and Schmidt [34].

As a next step, we are currently considering the strange meson sector[35] within the Dynamical AdS/QCD model. For a future challenge we also want to introduce finite temperature and calculate the spectrum as done in ref. [36] and compare to recent Lattice results[37].

We acknowledge partial support from CAPES, FAPESP and CNPq.

# REFERENCES

- J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231
- I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B 578, 123 (2000)
- I. R. Klebanov and M. J. Strassler, JHEP 0008, 052 (2000)
- J. M. Maldacena and C. Nunez, Phys. Rev. Lett. 86, 588 (2001)
- J. Erdmenger, N. Evans, I. Kirsch and E. Threlfall, Eur. Phys. J. A 35, 81 (2008)
- M. Bianchi and W. de Paula, arXiv:1003.2536 (2010)
- J. Polchinsky and M. J. Strassler, Phys. Rev. Lett. 88 (2002) 031601.
- H. Boschi, N. Braga and H. Carrion, Eur. Phys. J. C 32 (2004) 529; Phys. Rev. D 73 (2006) 047901;
   G. F. de Téramond and S. J. Brodsky, Phys. Rev. Lett. 94 (2005) 0201601; J.Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95(2005) 261602; L. Da Rold and A. Pomarol, Nucl. Phys. B 721 (2005) 79; S. J. Brodsky and G. F. de Téramond, Phys. Rev. Lett. 96 2006) 0201601; Phys. Rev. D 77 (2008) 056007; C.A. Ballon Bayona, H. Boschi-Filho and N. Braga, JHEP 0809 (2008) 114;
- A. Karch, E. Katz, D.T. Son and M.A. Stephanov, Phys. Rev. D 74 (2006) 015005.
- S. Kuperstein and J. Sonnenschein, JHEP 11 (2004) 026; M. Kruczenski, L.A.P. Zayas, J. Sonnenschein and D. Vaman, JHEP 06 (2005) 046;
   O. Andreev and V.I. Zakharov, Phys. Rev. D 74 (2006) 025023; Phys. Rev. D 76 (2007) 047705;
   J. P. Shock, F. Wu, Z. Xie, JHEP 03 (2007) 064.
- H. Forkel, M. Beyer and T. Frederico, JHEP 0707 (2007) 077.
- 12. J. Maldacena, Phys. Rev. Lett. 80 (1998) 4859.
- S.J.Rey and J.T.Yee, Eur. Phy. J. C 22 (2001) 379.
- H. Forkel, Phys. Rev. D 78 (2008) 025001.
- 15. C. Csaki and M. Reece, JHEP **05** (2007) 062.
- B. Batell and T. Gherghetta, Phys. Rev. D 78 (2008) 026002.
- W. de Paula, T. Frederico, H. Forkel and M. Beyer, Phys. Rev. D 79 (2009) 075019; PoS LC2008 (2008) 046.
- U. Gürsoy, E. Kiritsis and F. Nitti, JHEP 0802(2008) 019; JHEP 0802(2008) 032.
- E. Katz, A. Lewandowski and M.D. Schwartz, Phys. Rev. D 74 (2006) 086004.
- 20. A. Vega and I. Schmidt, Phys. Rev. D 78 (2008)

- 017703.
- P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau, and S. Nicotri, Phys. Rev. D 78 (2008) 055009.
- W. de Paula and T. Frederico, arXiv:0908.4282v1 (2009).
- 23. E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.
- C. Amsler et al. (Particle Data Group), Phys. Lett. B 667 (2008) 1.
- H. Grigoryan and A. Radyushkin, Phys. Rev. D 76 (2007) 095007.
- S. J. Brodsky and G. F. de Téramond, Phys. Rev. D 78 (2008) 025032.
- E. M. Aitala et al. (Fermilab E791 collaboration),
   Phys. Rev. Lett. 86 (2001) 770.
- H. Muramatsu et al. (CLEO collaboration), Phys. Rev. Lett. 89 (2002) 251802.
- I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001.
- D. V. Bugg, Eur. Phys. Jour. C 47 (2006) 57; AIP Conf. Proc. 1030 (2008) 3.
- I. Bediaga, F. Navarra and M. Nielsen, Phys. Lett. B 579 (2004) 59.
- G. F. de Téramond and S. J. Brodsky, Phys. Rev. Lett. 102 (2009) 081601.
- A. Vega, I. Schmidt, T. Branz, T. Gutsche and V. Lyubovitskij, Phys. Rev. D 80 (2009) 055014.
- A. Vega and I. Schmidt, Phys. Rev. D 80 (2009) 055003.
- 35. K.S.F.F. Guimarães, W. de Paula, I. Bediaga, A.Delfino, T. Frederico, A.C.dos Reis and L. Tomio, in progress.
- A.S. Miranda, C.A. Ballon Bayona, H. Boschi-Filho and N. Braga JHEP 0911 (2009) 119; P. Colangelo, F. Giannuzi and S. Nicotri, Phys. Rev. D 80 (2009) 094019.
- 37. M. Panero, Phys. Rev. Lett.  ${\bf 103}~(2009)~232001.$